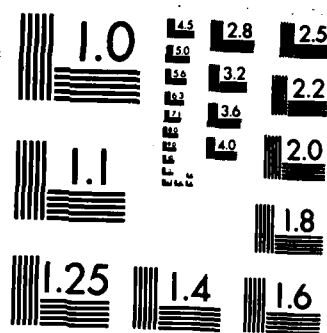MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

UNCLASS
SECURITY CLASS

AD-A154 912

	READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUM SSION NO. R-069	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Basic Formulae for the Computation of the Deflection of the Vertical from Astronomical Star Observations	5. TYPE OF REPORT & PERIOD COVERED
	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Dr. Angel A. Baldini	8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS US Army Engineer Topographic Laboratories Fort Belvoir, VA 22060-5546	12. REPORT DATE March 1985
	13. NUMBER OF PAGES
14. MONITORING AGENCY NAME & ADDRESS(If different from Controlling Office)	15. SECURITY CLASS. (of this report) Unclassified
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release
Distribution unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)
new concepts for determining the astro-geodetic deflection of the vertical
effect of errors in evaluating the deflection components ξ and η as well as the
polar migration, are considered.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

In this report we deal with new concepts for determining the astrogeodetic deflection of the vertical, in the sense that we determine the ξ and η deflection components completely independently of the knowledge of the astronomic coordinates, latitude and longitude, hence reducing the time and cost of field work. The first part of this paper is devoted to finding a relationship between the deflection components with respect to a star or stars positions. We have found that the ξ and η components are related to either the difference between the astronomic and geodetic star zenith distances; azimuths; parallactic angles, or are

DD FORM 1473   EDITION OF 1 NOV 65 IS OBSOLETE   UNCLASSIFIED

based on a star rate of change in azimuth and zenith distance with respect to time. Except for the last one, star pairs must be considered. In the second part of this paper, we must consider a method that deals with the solution for the deflection components ξ and η from observation of star pairs, as function of the astronomic and geodetic zenith distances. The effect of errors in evaluating the deflection components ξ and as η well as the polar migration, are considered.

Accession For

NTIS GRA&I ☑

DTIC TAB ☐

Unannounced ☐

Justification

By

Distribution/

Availability Codes

Avail and/or
Special

Dist

A-1

BASIC FORMULAE FOR THE COMPUTATION OF THE DEFLECTION OF THE VERTICAL FROM ASTRONOMICAL STAR OBSERVATIONS.

Dr. Angel A. Baldini
Research Institute
U.S. Army Engineer Topographic Laboratories
Fort Belvoir, VA 22060

BIOGRAPHICAL SKETCH

Dr. Angel A. Baldini joined ETL's predecessor organization in 1960. From 1957-1960 he was associated with the Georgetown University Observatory, Washington, D.C. Prior to 1957, he was Professor and Head, Department of Geodesy, La Plata University, Argentina. AT ETL, Dr. Baldini worked primarily in the field of astro-geodesy. Since 1974 he has been senior scientist in the Center for Geodesy, Research Institute, ETL. He authored 40 reports and papers since 1963 and presented results thereof at 25 Army, national, and international meetings. He received an Army Research Development achievement award in 1969. He has been a member of the American Geophysical Union since 1960.

ABSTRACT

In this report we deal with new concepts for determining the astro-geodetic deflection of the vertical, in the sense that we determine the ξ and η deflection components completely independently of the knowledge of the astronomic coordinates, latitude and longitude, hence reducing the time and cost of field work. The first part of this paper is devoted to finding a relationship between the deflection components with respect to a star or stars positions. We have found that the ξ and η components are related to either the difference between the astronomic and geodetic star zenith distances; azimuths; parallactic angles, or are based on a star rate of change in azimuth and zenith distance with respect to time. Except for the last one, star pairs must be considered. In the second part of this paper, we consider a method that deals with the solution for the deflection components ξ and η from observation of star pairs, as function of the astronomic and geodetic zenith distances. The effect of errors in evaluating the deflection components ξ and η as well as the polar migration, are considered.

INTRODUCTION

The deflection of the vertical at a point is the angle which the physical plumbline at this point makes with the corresponding normal to the ellipsoid.

Consider, figure 1, a sphere of a unit radius with its center at the observation station. Let Z_a be the astronomical zenith and Z_g be the geodetic zenith. Let S be the point on the sphere where the line of sight to a star intersects the unit sphere at a certain instant of time.

85-5-15 055

5 15 055

Let Z and $\quad$ be the zenith distance of the star with respect to Z_a and Z_g respectively. The point of correspondence to the direction of the north pole, which has the zenith distance $(90°-0)$ and $(90°-B)$, with respect to Z_a and Z_g; t and $\quad$ are the star's hour angles with respect to the astronomic and geodetic meridians respectively. P_g and P_a, are the star's parallactic angles with respect to Z_a and Z_g respectively, so that according figure 1:

$$Z_a \; S \; Z_g = P_g - P_a$$

Let Z_g T be an arc of a small circle at which S is the pole, then ST $= S Z_a$. Since $Z_a Z_g$ is a small circle of arc, we may assume that $Z_a T Z_g$ is a plain triangle, with a right angle at T. Let
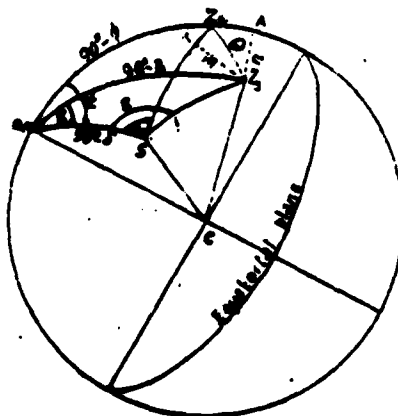
$$y = Z - H$$
$$m = Z_g T$$

Figure 1. The unit sphere illustrating the deflection of the vertical with respect to a star position.

Let θ be the angle between the astronomic and geodetic verticals. Because θ is small, we have:

$$\theta = \sqrt{y^2 + m^2}$$

The angle $Z_a \; S \; Z_g$ is also small, so we have:

$$\theta = \sqrt{(Z-Z')^2 + (\beta-\beta')^2 \sin^2 Z} \quad . \tag{1}$$

Let us draw the arc Z_g A from Z_g perpendicular to the astronomic meridian. We then have

$$A \; Z_a = \xi - \text{the component of the total deflection of the vertical along the meridian.}$$

$$A \; Z_g = \eta - \text{the component of the total deflection of the vertical along the prime vertical.}$$

In the rectangular spherical triangle APZ_g we have for the ξ and η deflection components:

$$\sin \eta = \cos \delta \sin(t - t') \tag{2}$$

$$\sin \eta \cos 90 = \cos(\phi - \xi) \sin \delta - \sin(\phi - \xi) \cos \delta \cos(t - t')$$

For the small angles η and $(\xi-\zeta)$ we get

$$\eta \cos(\lambda - \zeta) = \eta$$
$$\phi - \xi = \beta \tag{3}$$

Let us see how the deflection components ξ and η, with respect to a star position, can be defined independently of astronomic coordinates.

A Z_a S is the star's astronomic azimuth, and KZ_gA is the geodetic azimuth. The astronomic azimuth is represented by (Fig. 2)
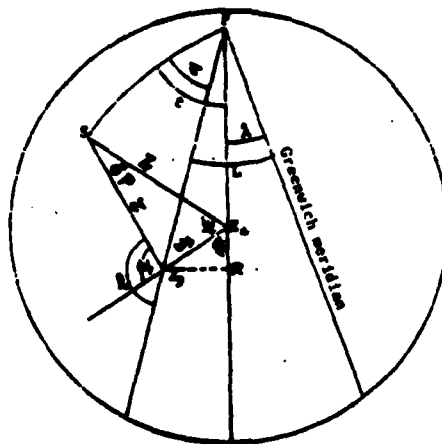
$$A = U_1 + U_2 \tag{4}$$

Figure 2. Relationship between Astronomic and Geodetic Azimuth.

and the geodetic azimuth by

$$\alpha = W_1 + W_2 . \tag{5}$$

The difference in azimuth consists of two parts

$$A - \alpha = (U_1 - W_1) + (U_2 - W_2) \tag{6}$$

of these two parts the second (U_2-W_2) is always very small. Using the equation

$$\sin B \cos c = \cos C \sin A + \sin C \cos A \cos b \tag{7}$$

on the spherical triangle $Z\ Z_a\ Z_g$ figure 1, we obtain:

$$\sin(\beta_g - \beta_a)\cos \chi = \cos U_1 \sin W_1 - \sin U_1 \cos W_1 \cos \theta . \tag{8}$$

By treating 0 as small, we obtain

$$\sin(\beta_g - \beta_a)\cos \chi = \sin(W_1 - U_1) \tag{9}$$

hence:

$$W_1 - U_1 = (\beta_g - \beta_a) \cos \chi \tag{10}$$

For the second part (U_2-W_2) we make a quite similar development, with regard to the spherical triangle $Z_a\ Z_g P$. Taking again equation (7), we have

$$\sin(\xi - z)\sin B = -\cos(A-U_1)\sin W_2 + \sin(A-U_1)\cos W_2 \cos \theta \tag{11}$$

$$\sin(\xi - \zeta)\sin B = \sin[A - (U_1 + W_2)]$$

3

and hence:

$$(t-\tau)\sin B = A - (U_1 + W_2)$$
$$- \eta \tan B = A - (U_1 + W_2) \tag{12}$$

From equation (4) we have:

$$A - u_1 = u_2$$

hence

$$u_2 - W_2 = (t-\tau)\sin B. \tag{13}$$

Replacing in equation (6) the values of $(W_1 - U_1)$ from equation (10) and $(U_2 - W_2)$ from equation (13), we obtain:

$$A - \alpha = (\ell_e - \ell_g)\cos Z + (t-\tau)\sin B. \tag{14}$$

With respect to the Greenwich meridian we have from figure 2:

$$t + \lambda = \tau + L. \tag{15}$$

Where , L are the astronomic and geodetic longitudes respectively taken positive to the west.

It follows then that:

$$t - \tau = -(\lambda - L). \tag{16}$$

In the triangle AZP, we have:

$$\eta = AZ = (t-\tau)\cos B = -(\lambda - L)\cos B. \tag{17}$$

Substituting in equation (14), the value of (   ) shown in equation (17), we obtain

$$A - \alpha = (\ell_e - \ell_g)\cos Z - \eta \tan B. \tag{18}$$

This equation links the deflection of the prime vertical component, to the astronomic and geodetic azimuth and parallatic angle of a star position.

Deflections of the Vertical as a Function of the Absolute Deflection of the Vertical 0 and its Astronomic Azimuth U_2.

We determine the two components ξ and η of the deflection of the vertical to the north and east, from

$$\xi = \theta \cos U_2 \qquad\qquad \eta = \theta \sin U_2 \tag{17}$$

For the triangle $S\ Z_a\ Z_\xi$ applying the trigonometrical equation

$$\sin a \cos B = \sin c \cos b - \cos c \sin b \cos A \tag{18}$$

4

We have:

$$A = \delta P \qquad\qquad a = \theta$$
$$\qquad\qquad\qquad b = \mathcal{z}$$
$$B = \mathcal{U}, \qquad\qquad c = Z$$

hence, it follows

$$\sin\theta\cos\mathcal{U}_1 = \sin Z\cos\mathcal{z} - \cos Z\sin\mathcal{z}\cos\delta P \tag{19}$$

By treating δP as small, we obtain

$$\theta\cos\mathcal{U}_1 = Z - \mathcal{z} \tag{20}$$
$$\cos\mathcal{U}_1 = 1 - 2.\sin^2\tfrac{1}{2}\mathcal{U}_1$$

hence

$$\sin\tfrac{1}{2}\mathcal{U}_1 = \sqrt{\frac{\theta - (Z-\mathcal{z})}{2\theta}} \tag{21}$$

From equation (4) we obtain

$$\mathcal{U}_2 = A - \mathcal{U}_1 \tag{22}$$

from which equations (17) become

$$\xi = \theta\cos(A-\mathcal{U}_1)$$
$$\eta = \theta\sin(A-\mathcal{U}_1) \tag{23}$$

and the absolute deflection of the vertical

$$\theta^2 = \xi^2 + \eta^2 \tag{24}$$

η is derived by equation (15)

$$\eta = (t-\tau)\cos B$$

and θ from equation (3)

$$\theta^2 = \sqrt{(Z-\mathcal{z})^2 + (P_2 - P_3)^2\sin^2 Z}$$

and hence

$$\xi = \sqrt{(Z-\mathcal{z})^2 + (P_2-P_3)^2\sin^2 Z - (t-\tau)^2\cos^2 B} \tag{25}$$

The sign to be considered is derived from equation (23).

If the deflection component η is derived from equation (16), we may obtain the deflection component ξ from

$$\xi = \sqrt{(Z-\mathcal{z})^2 + (P_2 - P_3)^2\sin^2 Z - [(A-\alpha) - (P_2-P_3)\cos Z]^2\cot^2 B} \tag{26}$$

5

Deflections of the Vertical as Function Either the Astro and Geodetic Zenith Distances or Parallactic Angles.

From equation (23) we have for evaluating   :

$$\xi = \theta \cos(A-u_1) = \theta \cos u_1 \cos A + \theta \sin u_1 \sin A \qquad (27)$$

From figure (2) we get:

$$\theta \cos u_1 = \sin Z \cos \mathcal{Z} - \cos Z \sin \mathcal{Z} \cos(P_g - P_a) \qquad (28)$$

Because $(P_g - P_a)$ is small, we assume

$$\cos(P_g - P_a) = 1$$

hence:

$$\theta \cos u_1 = Z - \mathcal{Z} \qquad (29)$$

and

$$\theta \sin u_1 = (P_g - P_a) \sin \mathcal{Z} \qquad (30)$$

Inserting the values shown in equations (29) and (30) into equation (27), we finally obtain:

$$\xi = -(\mathcal{Z}-Z) \cos A \pm (P_g - P_a) \sin \mathcal{Z} \sin A \qquad (31)$$

For the deflection component in the prime vertical we have:

$$n = \theta \sin(A-u_1) = \theta \sin u_1 \cos A - \theta \cos u_1 \sin A \qquad (32)$$

Substituting in this equation the values shown in equations (29) and (30), we get

$$n = (Z-\mathcal{Z}) \sin A \mp (P_g - P_a) \sin Z \cos A \qquad (33)$$

This equation is deducted from figure (2). Let us see if the sign of n is correct. From figure (2) we have

$$n = (t-\tau) \cos B$$

and

$$t-\tau = L-\lambda$$

hence equation (33) must be rewritten as follows:

$$n = (\mathcal{Z}-Z) \sin A \pm (P_g - P_a) \sin Z \cos A \qquad (34)$$

Collecting now the basic equations that express the components ξ and n of the deflections of the vertical in terms of the astronomic and geodetic coordinates of a star position, we have

$$n \tan B = \alpha - A + (P_a - P_g) \cos Z \qquad (18)$$

$$\xi = -(\mathcal{Z}-Z) \cos A + (P_g - P_a) \sin \mathcal{Z} \text{ si } A \qquad (31)$$

$$n = (\mathcal{Z}-Z) \sin A + (P_g - P_a) \sin Z \cos A \qquad (34)$$

Equation (18) makes possible the calculation of the amount of the
deflection component in the prime vertical on the basis of the
derived astronomic azimuth and parallactic angle (independent of the
station astronomic coordinates) and the calculation of the
corresponding values in reference of the geodetic system. Equations
(31) and (34) evaluate ξ and η as function of zenith distances.
and parallactic angles. We can combine equations (18), (31) and (34)
involving the knowledge of only one term, either $(\zeta-Z)$, (P_g-P_a) or
$(\alpha-A)$. Eliminating $(\zeta-Z)$ between equations (31) and (34), we obtain:

$$\xi \sin\alpha + \eta \cos\alpha = (P_g - P_a) \sin\zeta \qquad (35)$$

and by eliminating $(P_g - P_a)$, we get:

$$-\xi \cos\alpha + \eta \sin\alpha = \zeta - Z \qquad (36)$$

Because of the small values of   and   , it makes no difference
whether the astronomic or geodetic azimuth is used. To obtain ξ
and η as a function of azimuth, we eliminate $(P_g - P_a)$ between
equation (35) and equation (18), thus we obtain:

$$\xi \sin\alpha + \eta(\tan\beta \tan\zeta + \cos A) = (\alpha - A) \tan\zeta \qquad (37)$$

FINDING THE ASTROGEODETIC DEFLECTION OF THE VERTICAL

We shall treat exclusively the methods of determining the
astrogeodetic deflection of the vertical; for this depends only upon
the actual selection of which equation among those shown in (18),
(31), (34), (35), (36), or (37) is to be solved. We consider in this
report a method for the solution that deals with equation (36):

$$\zeta - Z = -\xi \cos\alpha + \eta \sin\alpha$$

The other methods for the other equations shall not be considered in
this report. It shall be published in sequal reports.

FIRST METHOD - DEFLECTION OF THE VERTICAL BY EQUAL ALTITUDES OF STAR PAIRS.

Let the altitude or zenith distance of two stars of known coordinates
be observed at the instant when they cross the intersection of the
center vertical and horizontal lines. Let Z be the constant zenith
distance of observation of an individual star's pair; L_1 and L_2, be
the horizontal scale readings and let T_1 and T_2 be the respective
time of observation. The time system is synchronized from radio
signals transmitted by a well known time service station. Stellar
positions must be taken from the Four Fundamental Catalogue (FK4).

In the triangle $S_1 P S_2$ applying the formula of cosine, we have:

$$\cos S = \sin\delta_1 \sin\delta_2 + \cos\delta_1 \cos\delta_2 \cos\tau \qquad (38)$$

7

where

$$\Gamma = (T_2 - T_1)(1 + c) - (\alpha_2 - \alpha_1) \qquad (39)$$

c is a constant the value of which is:

$$c = 0$$

if the observation time is regulated to sidereal time, and

$$c = 1.002737909$$

when mean time is used.

In the triangle $S_1 Z S_2$ using the formula of cosine in the form:

$$2 \cos c = \cos(a+b)(1 - \cos C) + \cos(a-b)(1 + \cos C) \qquad (40)$$

and since

$$Z = Z_1 = Z_2$$

we have

$$2 \cos S = \cos 2Z (1 - \cos \Delta) + 1 + \cos \Delta \qquad (41)$$

hence

$$\cos 2Z = \frac{2 \cos S - (1 + \cos \Delta)}{1 - \cos \Delta} \qquad (42)$$

This formula can be simplified. Observing that we can put:

$$\cos 2Z = 1 - 2 \sin^2 Z$$
$$2 \cos S = 2 - 4 \sin^2 \tfrac{1}{2} S \qquad (43)$$
$$\cos \Delta = 1 - 2 \sin^2 \tfrac{1}{2} \Delta$$

we get

$$\sin Z = \frac{\sin \tfrac{1}{2} S}{\sin \tfrac{1}{2} \Delta} \qquad (44)$$

which is the required formula for computing the zenith distance.

EFFECT OF ERRORS IN EVALUATING Z

To discuss the effect of errors in the data upon the zenith distance by observation of a star pair, and hence also the conditions most favorable to accuracy, we have by differentiating equation (53)

$$dZ = \frac{1}{2} \frac{\tan Z}{\tan \tfrac{1}{2} \Delta} dS - \frac{1}{2} \frac{\tan Z}{\tan \tfrac{1}{2} \Delta} (dL_2 - dL_1) \qquad (45)$$

8

From equations (47) and (48), we obtain by differentiation,

$$dS = \frac{\cos\delta_1 \cos\delta_2 \sin\sigma}{\sin S}(dT_2 - dT_1) \qquad (46)$$

from which equation (54) becomes

$$dZ = \frac{1}{2}\frac{\tan Z}{\tan\frac{1}{2}\delta}\cos\delta_1\cos\delta_2\frac{\sin\sigma}{\sin S}(dT_2 - dT_1) - \frac{1}{2}\frac{\tan Z}{\tan\frac{1}{2}D}(dL_2 - dL_1) \qquad (47)$$

This equation shows that, in order to reduce the effect of errors on T_2, T_1, L_1, and L_2 as much as possible, we must make $\tan\frac{1}{2}(L_2 - L_1)$ as great as possible, and hence $= L_2 - L_1$, the difference of the azimuth, should be as close to 180°, and with not large zenith distance. Furthermore, the first term in the right side of equation (56) can be reduced by choosing one star to the north with large declination and azimuth less than 30°, with respect to the meridian plane, and the other star satisfying the condition that,

$$(L_2 - L_1) = (A_2 - A_1) > 150° \leq 180°$$

In that condition we have that the great circle S is greater than σ, so $\sin\sigma/\sin S$ is less than unity.

Equation (52) allows us to compute the zenith distance Z, which is to be used in equation (36). The geodetic zenith distance must be computed. Considering its evaluation. Let the time be related to the Greenwich mean time or universal time (U-T).
Let

      L = geodetic longitude, positive toward west
      B = geodetic latitude
      θ_o = sidereal time at 0^h U-T
      τ = star geodetic hour angle
      α, δ = right ascension and declination of a star
      χ = star geodetic zenith distance

The hour angel τ is computed from

$$\tau = 1.0027379094 \, \overline{UT} + \theta_o - (\alpha + L) \qquad (48)$$

and the zenith distance from:

$$\cos\chi = \sin B \sin\delta + \cos B \cos\delta \cos\tau \qquad (49)$$

To satisfy equation (36) we must know the geodetic azimuth, which is computed from

$$\tan\alpha = \frac{\sin\tau}{\sin B \cos\tau - \cos B \tan\delta} \qquad (50)$$

9

The astronomic zenith distance Z, as computed from equation (52), refers to the instantaneous vertical position while the geodetic vertical refers to the mean position at the date of the observation. Hence we proceed as more convenient to reduce the geodetic coordinates to the instantaneous pole position. The International Latitude Service continuously observes the variation of latitude at several stations and thus determines the motion of the pole, and publishes the coordinates of the instantaneous pole and date for correcting observed values. The geodetic coordinates reduced to the instantaneous pole position at the date of field observations is accomplished by means of the equations

$$B_o = B + x \cos L + y \sin L$$

$$L_o = L + (y \cos L - x \sin L) \tan B - y \tan \phi_{Gr}$$

Here we take longitude positive to the west. Hence B_o and L_o, are the values to be used in equations (57), 58 and (59), to compute z and α. The term containing (latitude of Greenwich) is usually omitted.

CONCLUSIONS AND RECOMMENDATIONS

A great need exists for the determination of the deflection of the vertical. The equations presented and illustrated in this paper can be advantageously employed. The conclusions to be drawn are the following:

The classical method of determining deflection of the vertical components are in terms of the geographic coordinates:

$$\xi = \phi - B$$
$$\eta = (\lambda - L) \cos B$$

Therefore for the determination of the deflection of the vertical there is only one means, that is the comparison of astronomic and geodetic latitudes and longitudes.

The equations we have derived for the computation of the deflection of the vertical components do not require knowledge of the astronomic coordinates latitude or longitude, hence reducing the time and cost of field work. Three different procedures have been found to compute the deflection components. Equation (35) gives the components as function of astronomic and geodetic parallatic angles. These angles can be obtained by observing star pairs on different, arbitrarily chosen, fixed vertical planes. The star image transit times over several vertical reticle lines are to be reduced to the central vertical line. For further details see reference 2.

Equation (36) gives the deflection components as function of the astronomic and geodetic star zenith distance. A method for the computation of the absolute zenith distance is included in this report. It can also be derived by observing a set of stars in a fixed almucantar. A method of independent equations to obtain the alumcantar zenith distance is shown in reference 3. The determination of zenith distance by this method is less prone to observational errors than it is with the determination of latitude or longitude. Equation (37) gives the deflection components as function of astronomic and geodetic azimuth. Absolute astronomic azimuth can be computed as indicated in reference 4.

REFERENCES

1 BALDINI, A. A.   (1983) Applicability of Stellar Rate for
                                 Astronomic Position Determination.
                                 Presented at ACSM/ASP Fall Conven-
                                 tion, Salt Lake City, Utah.

2   (1981) Techniques for Improvement of
                                 Astronomic Positioning in the Field.
                                 Presented at the ACSM-ASP Conven-
                                 tion, Washington, D.C.

3   (1963) Method of Independent Equations for
                                 Determining Latitude, Clock Correc-
                                 tion and Zenith Distance. Presented
                                 at the XIII General Assembly of the
                                 International Union of Geodesy and
                                 Geophysics, Berkley, California.

4   (1966). New Method for Determining Azimuth
                                 and Latitude Independent of Time and
                                 Zenith Distance. GIMRADA Research
                                 Notes, 17 US Army Engineer, Fort
                                 Belvoir, VA.

END

FILMED

7-85

DTIC